

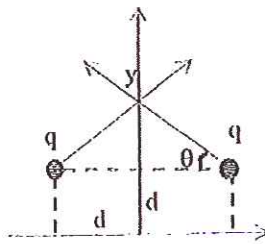
Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name:

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Two identical charges of $+q$ are placed at $x = \pm d$ on the x -axis. Determine the position and the value of the maximum electric field on the y -axis. (The constant in Coulomb's law is $\frac{1}{4\pi\epsilon_0}$)



Solution:

$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

$E_x = 0$ because of symmetry.

$$E_y = E \sin \theta = \frac{1}{4\pi\epsilon_0} \frac{2q}{(d^2 + (y-d)^2)} \cdot \frac{(y-d)}{\sqrt{d^2 + (y-d)^2}} = \frac{q}{2\pi\epsilon_0} \cdot \frac{(y-d)}{(d^2 + (y-d)^2)^{3/2}}$$

$$E = \frac{q}{2\pi\epsilon_0} \cdot \frac{(y-d)}{(d^2 + (y-d)^2)^{3/2}}$$

To find maximum of the field we use $\frac{dE}{dy} = 0$.

$$\text{This gives } (d^2 + (y-d)^2)^{3/2} - 3(y-d)^2(d^2 + (y-d)^2)^{1/2} = 0$$

$$2(y-d)^2 = d^2 \text{ or } y = d \pm \frac{d}{\sqrt{2}} \text{ (position of max E-field)}$$

The max. value of E-field is: $E_{max} = \frac{q}{\pi\epsilon_0 d^2}$.

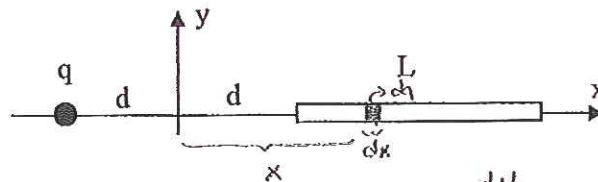
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A wire extends from $x = d$ to $x = d + L$. The wire has uniform charge density λ per unit length. Determine the sign and the magnitude of a point charge that must be placed to $x = -d$ such that the electric field at the origin is zero. Determine the magnitude of the point charge when the rod has infinite length in the $+x$ direction. Put $\frac{1}{4\pi\epsilon_0} = k$ in your calculations for simplicity.



$$dE = \frac{k \cdot dq}{x^2} = \frac{k \cdot \lambda dx}{x^2} \Rightarrow E = \int dE = k\lambda \cdot \int_d^{d+L} \frac{dx}{x^2} = -\frac{k\lambda}{x} \Big|_d^{d+L} = k\lambda \left[\frac{1}{d} - \frac{1}{d+L} \right]$$

$$\vec{E}_{\text{wire}} = -k\lambda \left[\frac{1}{d} - \frac{1}{d+L} \right] \hat{i}$$

$$\vec{E}_q = \frac{kq}{d^2} \hat{i}$$

$$\vec{E}_{\text{wire}} + \vec{E}_q = 0$$

$$q = \lambda d^2 \left[\frac{1}{d} - \frac{1}{d+L} \right] \hat{i}$$

the rod has infinite length:

$$E = k\lambda \int_d^{\infty} \frac{dx}{x^2} = -\frac{k\lambda}{x} \Big|_d^{\infty} = -k\lambda \left[\frac{1}{\infty} - \frac{1}{d} \right] = \left[\frac{k\lambda}{d} = E \right]$$

$$\left. \begin{aligned} \vec{E}_{\text{wire}} &= -\frac{k\lambda}{d} \hat{i} \\ \vec{E}_q &= \frac{kq}{d^2} \hat{i} \end{aligned} \right\} \frac{k\lambda}{d} = \frac{kq}{d^2} \Rightarrow q = \lambda d$$

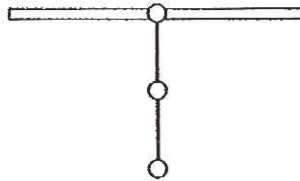
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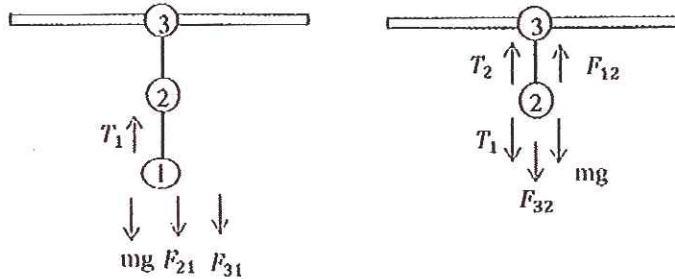
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Three identical particles, each of mass m and charge $+q$ are fixed at the ends and at the center of a rope of length $2d$ as shown in the figure. The rope is fixed from one end (including the particle at that end) to the ceiling. If the ratio of the tension in the upper half of the string to the tension in the lower half of the string is $3/2$, determine the charge q in terms of other quantities. Put $\frac{1}{4\pi\epsilon_0} = k$ in your calculation for simplicity. Leave the gravitational acceleration, g , symbolic (i.e. do not put its value into your calculations).



Solution: The forces acting on the 1st and 2nd particles are shown below.



$$T_1 = mg + F_{21} + F_{31},$$

$$T_2 = -F_{12} + T_1 + F_{32} + mg$$

Note that $F_{12} = F_{21} = F_{32} = F_{23} = \frac{kq^2}{d}$ and $F_{13} = F_{31} = \frac{kq^2}{4d^2}$. Substituting:

$$\frac{T_2}{T_1} = \frac{2mg + \frac{5kq^2}{4d^2}}{mg + \frac{5kq^2}{4d^2}} = \frac{3}{2}, \text{ Solving for } q: \frac{5kq^2}{4d^2} = mg. \quad q = \sqrt{\frac{4mgd^2}{5k}}$$

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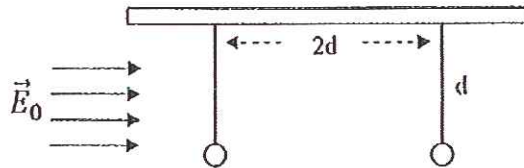
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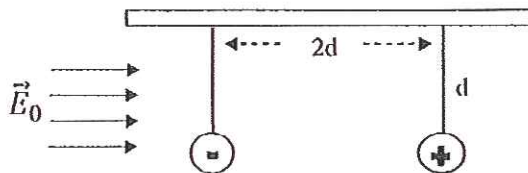
Two particles have the same mass m but opposite charges $\pm q$. The particles are fixed at the ends of two equal ropes of length d and suspended from two points separated by $2d$ as shown in the figure. Under a uniform constant electric field \vec{E}_0 applied externally in the $+x$ direction the particles stay vertical in equilibrium. Show on the figure which particle is $+q$ and which one is $-q$. Determine the magnitude of the electric field.

Then the magnitude of the electric field is reduced such that each rope makes an angle of 30° with the vertical. Determine the new distance between the particles.

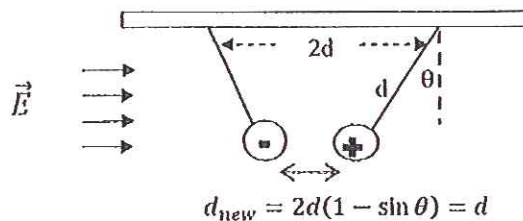
Put $\frac{1}{4\pi\epsilon_0} = k$ in your calculations for simplicity. Leave the gravitational acceleration, g , symbolic (i.e. do not put its value into your calculations).



Solution:



For the rope to stay vertical, the horizontal force must be zero: $qE_0 = \frac{kq^2}{4d^2}$ or $E_0 = \frac{kq}{4d^2}$.



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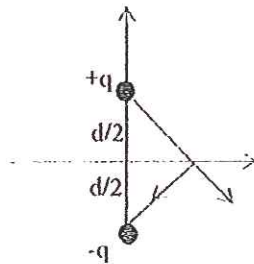
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Given two equal but opposite sign charges $\pm q$ with a fixed distance d between them. How would you put these charges in the x - y plane such that the electric field generated at any point on the x -axis is only in the $+y$ direction and its magnitude is maximum at $x = 0$? Draw your configuration and show that the conditions are satisfied. Put $\frac{1}{4\pi\epsilon_0} = k$ in your calculation for simplicity.

Solution:



The x -component of the electric field must vanish. That is the electric field generated by each charge must have equal but opposite directed components on the x -axis. This configuration is shown in the figure.

$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

$E_x = 0$ because of symmetry.

$$E_y = E \sin \theta = k \frac{2q}{(d^2 + x^2)} \cdot \frac{d/2}{\sqrt{d^2 + x^2}} = kq \cdot \frac{d}{(d^2 + x^2)^{3/2}}$$

$$E = kq \cdot \frac{d}{(d^2 + x^2)^{3/2}}$$

To find maximum of the field we use $\frac{dE}{dx} = 0$. This gives

$$\frac{dE}{dx} = kqd \cdot \frac{-3x}{(d^2 + x^2)^3} = 0 \text{ that is } x = 0.$$

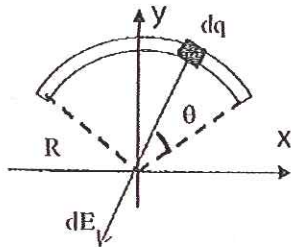
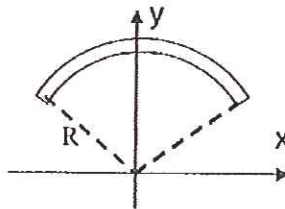
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A 120° circular arc with radius R is positioned with its center of curvature at the origin as shown in the figure. The arc is symmetric about the y -axis and it contains charge Q distributed uniformly. Determine the electric field at the origin. Put $\frac{1}{4\pi\epsilon_0} = k$ in your calculation for simplicity. (Hint: use the limits of angular integration from $\frac{\pi}{6}$ to $\frac{5\pi}{6}$)



Solution:

$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

Because of symmetry of the arc about the y -axis, the x -component of the electric field is zero.

$$dE_y = dE \sin \theta$$

$$dE = \frac{k dq}{R^2}$$

$$dq = \frac{Q}{2\pi R} R d\theta = \frac{3Q}{2\pi} d\theta$$

$$dE_y = \frac{3kQ}{2\pi R^2} \sin \theta d\theta$$

$$E_y = \frac{3kQ}{2\pi R^2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin \theta d\theta = \frac{3\sqrt{3}kQ}{2\pi R^2}$$